## Statistically Speaking Lecture Series

Sponsored by the Biostatistics Collaboration Center
Time-to-Event Analysis: A 'Survival' Guide
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## BCC: Biostatistics Collaboration Center

Who We Are
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## Biostatistics Collaboration Center (BCC)

Mission: to support investigators in the conduct of high-quality, innovative health-related research by providing expertise in biostatistics, statistical programming, and data management.

How do we accomplish this?

1. Every investigator is provided a FREE initial consultation of 1-2 hours, subsidized by FSM Office for Research. Thereafter:
a) Grants
b) Subscription
c) Re-charge (Hourly) Rates
2. Grant writing (e.g. developing analysis plans, power/sample size calculations) is also supported by FSM at no cost to the investigator, with the goal of establishing successful collaborations.

## BCC: Biostatistics Collaboration Center

## What We Do

- Many areas of expertise, including:
- Bayesian Methods
- Big Data
- Bioinformatics
- Causal Inference
- Clinical Trials
- Database Design
- Genomics
- Longitudinal Data
- Missing Data
- Reproducibility
- Survival Analysis

Many types of software, including:
sas


## BCC: Biostatistics Collaboration Center

Shared Statistical Resources

M Northustem Medicine ${ }^{\circ}$

NUCATS
Clinical and Translational Sciences Institute


Biostatistics Collaboration Center (BCC)

- Supports non-cancer research at NU
- Provides investigators an initial 1-2 hour consultation subsidized by the FSM Office of Research
- Grant, Hourly, Subscription


## Kibilitylab

Stanley Manne Children's Research Institute-
(II) Ann \& Robert H. Lurie Children's Hospital of Chicago*

Biostatistics Research Core (BRC)

- Supports Lurie Children's Hospital affiliates
- Provides investigators statistical support subsidized by the Stanley Manne Research Institute at Lurie Children's.
- Hourly


## BCC: Biostatistics Collaboration Center

## Shared Resources Contact Info

- Biostatistics Collaboration Center (BCC)
- Website: http://www.feinberg.northwestern.edu/sites/bcc/index.html
- Email: bcc@northwestern.edu
- Phone: 312.503.2288
- Quantitative Data Sciences Core (QDSC)
- Website: http://cancer.northwestern.edu/research/shared resources/quantitative data sciences/index.cfm
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- Phone:312.503.2288
- Biostatistics Research Core (BRC)
- Website: https://www.luriechildrens.org/en-us/research/facilities/Pages/biostatistics.aspx
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- Phone: 773.755.6328


## Time-to-Event Analysis: A 'Survival' Guide

Studies involving survival analysis

- Time to death in a breast cancer trial
- Time to hospitalization of children with pneumonia
- Time to recurrence of ovarian tumors
- Time to remission from depressive symptoms
- Time to cessation of postoperative opioids


## Objectives of survival analysis

- Estimate survival
- What is the probability of surviving 5 years post surgery?
- Compare survival between groups
- Are there differences in survival between treatment groups?
- Assess the relationship of covariates on the time-to-event
- How do clinical/behavioral characteristics affect survival?

Why do we care about time-to-event?

|  | Recurrence of <br> Tumor | No Recurrence of <br> Tumor |
| :--- | :---: | :---: |
| Treatment A | 25 | 40 |
| Treatment B | 30 | 35 |

Compare proportion of tumor recurrence between

$$
\begin{gathered}
\mathrm{OR}=0.73 \\
\mathrm{P} \text {-value }=0.38
\end{gathered}
$$

|  | Treatment A | Treatment B |
| :--- | :---: | :---: |
| Time to Recurrence (months) | $18.1 \pm 2.1$ | $8.5 \pm 3.4$ |

Why not traditional methods for time-to-event data?

- Incomplete information
- Not everyone experienced the event of interest


No time to event

## Why not traditional methods for time-to-event data?

- Compare mean time between groups?

|  | Treatment A | Treatment B |
| :---: | :---: | :---: |
| Time to Recurrence (months) | $18.1 \pm 2.1$ | $8.5 \pm 3.4$ |



## Why not traditional methods for time-to-event data?

- Compare proportion of events between groups
- Chi-square test, logistic regression?
- Ignores time
- Compare mean time between groups
- T-test, linear regression?
- Not normally distributed
- Ignores subjects without events




## Follow-up Time

- Defining time zero
- Time at which participants are considered at risk
- Enrollment into study
- Time of randomization
- Followed until
- Event occurs
- Study ends
- Participant is lost






## Censoring

- Right censoring (most common)
- Event occurs after a certain time point, but unknown how long after
- Study ends
- Lost to follow-up
- Subject withdraws


## Censoring

- Left censoring
- Event occurs before a certain time point, but unknown how much earlier



## Censoring

- Interval censoring
- Only know that the event occurred within a certain interval of time



## Censoring

- Methods require assumption that censoring is independent of event process
- Patients censored representative of patients still at risk
- Knowledge of censoring provides no information of survival at future time
- Not independent?
- Follow participants until death from lung cancer
- Subject dies from another cancer


## Basic Quantities

## Survival Function



Time 0: Randomization of treatment

## Basic Quantities

## Survival Function

- Probability of an individual surviving beyond a specified time
- Never increases
- Defined up to the largest event time



## Methods - Estimation

## Estimating survival probability

- Kaplan-Meier (Product Limit Estimator)

Survival in patients with
Acute Myelogenous
Leukemia

- No assumptions about shape
- Takes censored observations into account
- Common for medical studies
- Estimated for each unique failure time





| 1.00 |
| :---: |
| 0.90 |
| 0.80 |
| 0.70 |






$$
1-1 / 8=0.875
$$











| $\begin{array}{ll} \frac{\lambda}{2} & 0.75 \\ \frac{\overline{0}}{0} & \\ \frac{0}{0} & \\ \frac{2}{0} & 0.50 \end{array}$ |  |  |  |  | Day | At risk | Events | Estimate |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | 7 | 8 | 1 | 0.875 |
|  |  |  |  |  | $12$ | 5 | 1 | 0.700 |
|  |  |  |  |  | 15 | 4 | 1 | 0.525 |
| $\stackrel{0.25}{ }$ |  |  |  |  | 20 | 2 | 1 | 0.263 |
| 0.00 |  |  |  | 20 | 22 | 1 | 1 | 0.000 |
|  | 5 | 10 | 15 |  |  |  |  |  |
| Survival Time (Days) |  |  |  |  |  |  |  |  |

How can we interpret?

- Estimate survival probability at specified time


How can we interpret?

- Estimate median failure time


## Example

## Estimating median failure time

Survival in patients with Acute Myelogenous Leukemia


What if last observation is censored?

| Day | At risk | Events | Estimate |
| :---: | :---: | :---: | :---: |
| 7 | 8 | 1 | 0.875 |
| 12 | 5 | 1 | 0.700 |
| 15 | 4 | 1 | 0.525 |
| 20 | 2 | 1 | 0.263 |
| 22 | 1 | 0 | -- |



Median failure time not always estimable

| Day | At risk | Events | Estimate |
| :---: | :---: | :---: | :---: |
| 7 | 8 | 1 | 0.875 |
| 12 | 5 | 1 | 0.700 |
| 15 | 4 | 1 | 0.525 |
| 20 | 2 | 0 | -- |
| 22 | 1 | 0 | -- |



## Basic Quantities

Hazard Function


## Basic Quantities

Hazard Function


## Basic Quantities

Hazard Function


## Basic Quantities

## Hazard Function

Survival Function


Hazard Function


## Basic Quantities

## Hazard Function



## Basic Quantities

## Cumulative Hazard Function



## Basic Quantities

Hazard Function

- Hazard Function
- Instantaneous failure rate at a specified time
- Measure of risk
- Non-negative
- Increasing, decreasing, or constant
- Cumulative Hazard
- Accumulation of risk up until a specified time
- Increasing or constant


## Methods - Estimation

## Cumulative Hazard function

Survival in patients with advanced lung cancer


## Methods - Inference

Comparing time-to-event between groups
Survival in patients with advanced lung cancer


[^0]
## Methods - Inference

## Comparing time-to-event between groups

Survival in randomized trial comparing treatments for ovarian cancer


[^1]
## Methods - Inference

Time to recurrence of colon cancer

- Adjust for another factor
- Few levels of factor
- Alternative to regression setting



## Methods - Inference

## Stratified Tests

## Stratify by Sex

Males
Treatment $+A+B$


Females


## Methods

## Assessing relationship of covariates on time-to-event

- Regression Models
- Cox Proportional Hazards Model
- Exponential(coefficient) = hazard ratio
- Hazard Ratio < 1: Reduction in hazard (risk of event) relative to reference group
- Hazard Ratio > 1: Increase in hazard (risk of event) relative to reference group

Interpretation of Hazard Ratio depends on how you code your variables!

## Cox Proportional Hazards Model

## Example

Survival in patients with advanced lung cancer

- Proportional hazards model

|  | Coefficient | Hazard Ratio | P-value |
| :---: | :---: | :---: | :---: |
| Male | 0.531 | 1.701 | 0.002 |

Sex + Male + Female

- Males have an increased risk of death
- There is a $70.1 \%$ increase in the expected hazard for males compared to females
- The expected hazard is 1.701 times higher in males compared to females



## Cox Proportional Hazards Model

## Example

|  | Coefficient | Hazard Ratio | P-value |
| :---: | :---: | :---: | :---: |
| Male | 0.513 | 1.671 | 0.002 |
| Age (years) | 0.017 | 1.017 | 0.065 |

- Holding age constant, being male increases the expected hazard by 67\%
- Holding sex constant, a one year increase in age is associated with a $2 \%$ increase in the expected hazard

|  | Coefficient | Hazard Ratio | P-value |
| :---: | :---: | :---: | :---: |
| Age (10 years) | 0.170 | 1.186 | 0.065 |

- A ten year increase in age is associated with a $20 \%$ increase in the expected hazard


## Cox Proportional Hazards Model

## Example

- Proportional hazards assumption
- Hazard functions are proportional over time

|  | Coefficient | Hazard Ratio | P-value |
| :---: | :---: | :---: | :---: |
| Male | 0.531 | 1.701 | 0.002 |

- Risk of death for males compared to females is constant over time
- Test for proportional hazards assumption
- Assess graphically
- Assess with interaction between variable and time
- Assess with test of proportionality (available in some statistical packages)


## Cox Proportional Hazards Model

## Example

|  | Coefficient | Hazard Ratio | P-value |
| :---: | :---: | :---: | :---: |
| Male | 0.531 | 1.701 | 0.002 |

- Test for proportional hazards

|  | P-value |
| :---: | :---: |
| Male | 0.117 |

- Does not violate proportional hazards assumption
- What if assumption is violated?
- Stratified analyses
- Interaction with time


## Other Topics

 Competing Risks- Subjects can 'fail' from more than one cause
- Prevent observation of event of interest
- Alter probability of an event of interest



## Other Topics

 Competing Risks- Subjects can 'fail' from more than one cause
- Prevent observation of event of interest
- Alter probability of an event of interest



## Other Topics

 Competing Risks- Example:
- Investigating death on dialysis
- Competing risk: receiving a kidney transplant



## Other Topics

Recurrent Events

- Multiple events occurring for one subject
- Examples:
- Recurrent tumors
- Recurrent episodes of disease


Survival Time (Days/Weeks/Years)

## Other Topics Frailty Models

- When survival outcomes are correlated among clustered individuals
- Model correlations between event times of same cluster
- Introduce random effects


Survival Time (Days/Weeks/Years)

## References

- Klein, JP and Moeschberger, ML. Survival Analysis: Techniques for Censored and Truncated Data. Second Edition. Springer, New York; 2003.
- R Development Core Team (2008). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0, URL http://www.Rproject.org.
- Other Survival Analysis Textbooks:
- Kleinbaum, DG. And Klein, M. Survival Analysis: A Self-Learning Text. Third Edition. Springer, New York; 2011.
- Moore, DF. Applied Survival Analysis Using R. Springer, New York; 2016.


## BCC: Biostatistics Collaboration Center

## Contact Us

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- Visit Our Website
- http://www.feinberg.northwestern.edu/sites/bcc/index.html

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Your feedback is important to us! (And helps us plan future lectures).

Complete the evaluation survey to be entered in to a drawing to win 2 free hours of biostatistics consultation.

Thank you for your participation in the BCC Statistically Speaking Lecture Series


[^0]:    M Northwestern Medicine
    Feinberg School of Medicine

[^1]:    M Northwestern Medicine
    Feinberg School of Medicine

